**Numeric Entry Exercise**

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1. Let A = {all 3-digit positive integers with the digit 1 in the ones place}, and let B = {all 3-digit positive integers with the digit 2 in the tens place}. How many elements are there in AU B?

Answer:

* Set A contains 9\*10\*1 = 90 numbers
  + 9 possibilities for hundreds place {1,2,….9}
  + 10 possibilities for tens place {0,1,2,….9}
  + 1 possibility for unit’s place {1}
* Set B contains 9\*1\*10 = 90 numbers
  + 9 possibilities for hundreds place {1,2,….9}
  + 1 possibility for tens place {2}
  + 10 possibilities for units place {0,1,2,….9}
* Set (A∩B) contains
  + 9 possibilities for hundreds place {1,2,….9}
  + 1 possibility for tens place {2}
  + 1 possibility for unit’s place {1}
* A∪B = A+B – (A∩B) = 90+90-9 = **171**

1. Yan needs $2.37 in postage to mail a letter. If he has 60-cent, 37-cent, 23-cent, 5-cent and 1-cent stamps, at least 10 of each, what is the smallest number of stamps he can use to get the exact postage he needs?

Answer:

* To reduce the number of stamps we must use the stamps with higher denomination and refrain from using stamps with lower denomination
* NOTE there isn’t any direct way to solve this question. Use trial and error:
* Case 1: 3 stamps of 60-cents
  + 60₵ - 3, 37₵ - 1, 23₵ - 0, 5₵ - 4, 1₵ - 0, {total = 3+1+4 = 8}
  + 60₵ - 3, 37₵ - 0, 23₵ - 2, 5₵ - 2, 1₵ - 1, {total = 3+2+2+1 = 8}
* Case 2: 2 stamps of 60-cents
  + 60₵-2, 37₵-3, 23₵-0, 5₵-1, 1₵-1, {total = 2+3+1+1 = 7} (best)
  + 60₵-2, 37₵-0, 23₵-5, 5₵-0, 1₵-2, {total = 2+5+2 = 8}
* Solution: **7**

1. A population of bacteria doubles every 2 hours. What is the percent increase after 4 hours?

Answer:

* Say we have 100 bacteria to begin with
* After 2 hours -> 100\*2 = 200
* After 4 hours -> 200\*2 = 400
* Percentage increase = [(400-100)/100] \* 100 = **300%**

1. Six chairs are placed in a row to seat six people. How many different seating arrangements are possible if two of the people insist on sitting next to each other?

Answer:

* Treat the 2 people as 1 unit
* We for arranging 5 units (6 people) there are 5! Ways
* Now the 2 people we considered 1 unit can also be arranged in 2! ways (they can interchange places)
* Total ways = 5! \* 2! = **240**

1. Let x, y and z be consecutive even integers. If the product of 3 and y is 32 more than the sum of x and z, what is the median of the numbers in set S = {x,y,z,2x,2y,2z}?

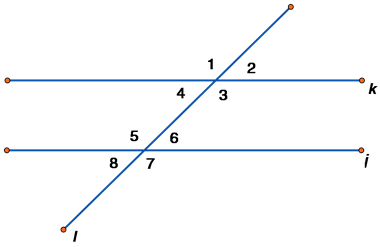
Answer:

* Let x, y, z be 2n-2, 2n, 2n+2
* 3y = x+z+32
* 3(2n) = 2n-2 + 2n+2 + 32
* n = 16, x=30, y=32, z=34
* median = (z+2x)/2 = **47**

1. If f(x)=x2 +3 and g(x) = x-5, evaluate f(g(9)).

Answer:

* g(9) = 9-5 = 4
* f(4) = 42+3 = **19**

1. A line intersects two parallel lines, forming eight angles. If one of the angles has measure a°, how many of the other seven angles are supplementary to it?

Answer:

* Assume angle 1 to be a°. Angle 2, 4, 6, 8 are supplementary to it.
* Solution: **4**

1. A rectangular box with length 22 inches, width 5 inches, and height 5 inches is to be packed with steel balls of radius 2 inches. What is the maximum number of balls that can fit into the box, provided that no balls should protrude from the box?

Answer:

* Incorrect Solution that yields correct answer ->
* Correct Solution (details cannot be shown here) ->
* Solution: **5**

1. Mary has d dollars to spend and goes on a shopping spree. First she spends 2/5 of her money on shoes. Then she spends ¾ of what’s left on a few books. Finally she buys a raffle ticket that costs 1/3 of her remaining dollars. What fraction of d is left?

Answer:

* Shoes = 2/5 \* d = 2d/5
* Left = d – 2d/5 = 3d/5
* Books = ¾ \* 3d/5 = 9d/20
* Left = d – 2d/5 – 9d/20 = 3d/20
* Raffle ticket = 1/3 \* 3d/20 = d/20
* Left = d – 2d/5 – 9d/20 – d/20 = 2d/20 = d/10
* Fraction left: **1/10**

1. Ten pounds of mixed nuts contain 50 percent peanuts. How many pounds of peanuts must be added so that the final mixture has 60 percent peanuts?

Answer:

* So the mixture contains 10\*(50/100) = 5 pounds peanuts
* So, after adding x pounds peanuts, total peanuts are 5+x pounds
* And the total mixture is 10+x pounds
* (5+x)/(10+x) = 60/100
* x = **2.5 pounds**

1. At John Adams High School, 120 students take programming, and 200 students take statistics. Of these, 50 students take both programming and statistics. An additional 80 students take neither programming nor statistics. If a student at this school is picked at random, what is the probability that he or she takes programming but not statistics?

Answer:

* Group A = students who take programming = 120
* Group B = students who take stats = 200
* Both = Students who take both = 50
* Neither = 80
* Total Students = Group A + Group B – (Both) + Neither
* Total Students = 120 + 200 – 50 + 80 = 350
* Students who take programming but not stats = Group A – Both = 120 - 50 = 70
* Probability = 70/350 = **1/5**

1. A hat contains the integers 1 to 100, inclusive. If a number is drawn at random from the hat, what is the probability that a multiple of 5 or a multiple of 8 is drawn?

Answer:

* Number of numbers from 1 to 100 which are divisible by 5 = 100/5 = 20
* Number of numbers from 1 to 100 which are divisible by 8 = 100/8 = 12.5 -> 12
* LCM(5,8) = 40
* Number of numbers from 1 to 100 which are divisible by both = 100/40 = 2.5 -> 2
* Total possibilities = 20 + 12 - 2 = 30
* Probability = 30/100 = **3/10**

1. A typing class in elementary school is divided into three groups. The Red Robins, with 6 students, has an average typing speed of 60 words per minute; the Blue Wax Bills, with 10 students, has an average typing speed of 45 words per minute; and the Gold Finches, with 16 students, has an average typing speed of 30 words per minute. What is the average (arithmetic mean) of the typing speeds, in words per minute, for the class?

Answer:

1. A charter company will provide a plane for a fare of $300 per person if there are between 50 and 100 passengers. If there are more than 100 passengers, then, for each additional passenger over 100, the fare will be reduced by $2 for every passenger. How much revenue will the company make if 120 passengers take the trip?

Answer:

* So we have a total of 20 extra passengers
* Reduction in cost = 20\*2 = $40 (for all passengers)
* Total revenue = 120\*(300-40) = **31200**

1. A solid white cube with an edge of 8 inches is painted. The cube is then sliced into 512 1- inch cubes. How many of these cubes have exactly 2 red faces?

Answer:

* Cubes on the edges have 2 faces painted red except the corner cubes (they have 3 faces painted red)
* 1 edge has 8 cubes of which 6 of them have two faces painted red.
* A cube has 12 edges.
* Solution: 12\*6 = **72**

1. When an elastic object, such as a coil spring or rubber band, is subjected to a force f, an increase in length, called a strain, occurs. Hooke’s law states that force f is directly proportional to strain s. Suppose that a coil spring has a natural length of 4 feet and that a force of 60 pounds stretches the length to 6 feet. What magnitude of force, in pounds, would stretch the spring to a length of 7 feet?

Answer:

* 60 pounds -> length increase = 6-4 = 2
* x pounds -> length increase = 7-4 = 3
* x = **90 pounds**

1. Rachel is hanging posters in her new apartment, which includes a bedroom, a living room, and a den. She has 7 different posters. Assuming that she plans to place exactly one poster in each of the three rooms, how many choices does she have?

Answer:

* 7C3 3P = **210**

1. A can contains ¼ pound of cashews. The can is then filled with a mixture that has equal weights of cashews, pecans and walnuts. If the final weight is 1 pound, what faction of the final nut mixture is cashew?

Answer:

* ¼ pound of cashews
* final weight is 1 pound
* mixture has weight = 1- ¼ = ¾
* weight of cashews in mixture is 1/3 \* ¾ = ¼ pound
* Total faction of cashew = (¼ + ¼)/1 = **½**

1. According to the table, for what value of x does g(f(x)) = -1?

Answer:

* 60 pounds -> 1080 miles
* 50 pounds -> x miles
* X = 50\*1080/60 = 900 miles
* Solution: 1080-900 = **180 miles**

1. For all values of x and y, let x\*y be defined as x\*y = 4xy/3. If 6\*a = 2, then a=

Answer:

* 6\*a = 4(6)(a)/3 = 2
* a = **1/4**

1. A hardware store owner finds that she can expect to sell n sets of wrenches per month if the price per set, in dollars, is: p(n) = 3000/(a+n) where a is a constant. If, according to this function, 25 sets of wrenches are sold in a month at $100 per set, how many sets can the owner expect to sell in a month if she raises the price to $200 per set?

Answer:

* n=25, p(n) = 100
* 100 = 3000/(a+25)
* a = 5
* Now, p(n) = 200
* 200 = 3000/(5+n)
* n = **10**

1. If the nth term of a sequence is given by the expression 2 x 4n-1, what is the unit’s digit of the 131st term in the sequence?

Answer:

* 2 x 4n-1 = 2 x 22n-2 = 22n-2+1 = 22n-1
* 131st term : 2261
* 21 = 2
* 22 = 4
* 23 = 8
* 24 = 6
* 25 = 2
* 261 % 4 = 1
* So units place is 21 = **2**

1. If f(x) = 3x2, at what x-co-ordinate do the graphs of f(x) and f(x-1) intersect?

Answer:

* f(x) = f(x-1)
* 3x2 = 3(x-1)2
* x = **1/2**

1. The faces of a cube are numbered with integers from 1 to 6 so that the sum of the numbers on opposite faces is 7. Thus, 1 is opposite 6, 2 is opposite 5 and 3 is opposite 4. If the cube is thrown on a flat surface so that 4 show on the top face, what is the probability that 6 is on the bottom face of the cube?

Answer:

* If 4 show on top, only 3 can be on the bottom face
* Probability = **0**

1. The average (arithmetic mean) of five positive even integers is 60. If p is the greatest of these integers, what is the greatest possible value of p?

Answer:

* Same as Statistics question 18 page 110
* Solution: **292**

1. Based on the information given in the table, what was the total number of ounces of toppings sold by the pizza parlor in February?

Answer

* (80)(16) + (165)(18) + (140)(28) = **8170**

1. (a x 2) + (a x 22) + (b x 23) + (b x 24) = 42. If a and b are positive integers, what is the value of ab?

Answer

* 5a + 24b = 42
* a=3, b=1 (trial & error)
* ab = **3**

1. For even integers x greater than 2, let \x/ be defined as the product of the even integers from 2 to x, inclusive. For example, \6/ = 2\*4\*6 = 48. If (\8/) / (\6/) = \a/, what is the value of a?

Answer

* (\8/) / (\6/) = (8\*6\*4\*2)/(6\*4\*2) = 8 = 4\*2 = \4/
* a = **4**

1. The table above shows the results of a survey of 300 people at an amusement part. Each person chose exactly one ride as his or her favorite. If 10 people were undecided, and x and y are both positive integers, what is the greatest possible value of y?

Answer

* 93+69+18+45+x+y+10 = 300
* x+y = 65
* for max. y x has to be min…. x = 1
* y = **64**

1. If f is a positive integer, fg > 0, and 6f+2g = 25, what is the sum of all possible values of g?

Answer:

* G also has to be positive
* All possible values of f and g {(1,9.5), (2,6.5), (3,3.5), (4,0.5)}
* Solution: 9.5+6.5+3.5+0.5 = **20**

1. A bag of dry concrete covers an area of 9 square feet. If only whole bags of dry concrete can be purchased, how many bags must be purchased to pave a sidewalk that is 3.5 feet wide and 225 feet long?

Answer:

* (3.5\*225)/9 = 87.5 -> **88 bags**

1. If t is a positive integer, and 18t is the cube of an integer, then what is the least possible value of t?

Answer:

* x = cube root (18t)
* 18t = 3\*3\*2\*t
* So t has to be 3\*2\*2 so the 18t = 3\*3\*3 \* 2\*2\*2
* t = **12**

1. The average (arithmetic mean) of 6 distinct numbers is 71. One of these numbers is -24, and the rest of the numbers are positive. If all of the numbers are even integers with at least two digits, what is the greatest possible value of any of the 6 numbers?

Answer:

* 10+12+14+16+x-24 = 71\*6
* x = **398**

1. On a number line, point D is 2/5 of the way from point C to point E and is located at -2. If C is at -10, what is the co-ordinate of point E?

Answer:

* DC = 2x = (-2) – (-10) = 8, x = 4
* DE = 3x = 12 = y – (-2), y = **10**

1. Stephanie, Damon and Karissa have been contracted to paint an office building that contains 72 rooms. If Stephanie paints half as many rooms as Karissa and 12 more than Damon, how many rooms does Karissa paint?

Answer

* S = x, K = 2x, D = x-12
* x+2x+x-12=72
* 2x = **42**

1. If x and y are real numbers, and , and , what is the smallest possible integer value of x+y?

Answer

* x = 6.25
* 3 < y < 7
* Solution: min(x+y) = 6.25+3.75 = **10**

1. One-third of the air in a tank is removed with each stroke of a pump. What percent of the original amount of air remains in the tank after five strokes?

Answer

1. Carlos paid $154.0 for two tickets to a concert. This price included a 25 percent handling fee for each ticket and a $2 transaction fee for the total sale. What was the price for a single ticket before the additional fee?

Answer

* Exclude transaction fee: 154 – 2 = 152
* Exclude handling fee: 152 \* (100/125) = 121.6
* Fee for each ticket = 121.6/2 = **60.8**

1. On January 1, 1993, Geraldine purchased a rare stamp for $ 300. The value of the rare stamp increased by 15 percent each year. If Geraldine decided to sell the stamp on January 1 of the first year in which its value had at least doubled since she purchased it, then in which year did Geraldine sell the stamp?

Answer

* Price after n years = 300 \* (115/100)n
* We want: 300 \* (115/100)n > 600
* So, n = 5
* Selling Year = 1993 + 5 = **1998**

1. Pierre receives a weekly allowance of $8, plus $3for each chore he completes during the week. Armand receives a weekly allowance of $6, plus $8 for each chore he completes during the week. Neither of them receives any other money. In a certain week, if they both complete the same number of chores, but Armand receives twice as much money as Pierre, then what is the total dollar amount that Armand receives in that week? (Disregard the $ sign)

Answer

* Let number of chores completed be x
* Pierre = 8 + 3x
* Armand = 6 + 8x
* 6 + 8x = 2 (8 + 3x)
* x = 5
* Armand = 6 + 8x = **46**

1. Three identical cubes, each with edges of length 8, are to be cut into a total of 384 identical rectangular solids of length 4. If the width and height of each solid are integers, what is the surface area of each solid?

Answer

* Equating volumes
* 3\*8\*8\*8 = 384 \* 4\*x\*y
* xy = 1; x=1;y=1
* Surface area: **18**

1. Segment PR is tangent to a circle with center O at point Q. If POR is a right triangle, QR=3, OR=5, and OP=20/3, then PQ =?

Answer

* Pythagoras: PR = sq. root [52 + (20/3)2] = 25/3
* PQ = PR – QR = 25/3 – 3 = **16/3**

1. Points J, K,L, M and N all lie on the same line. L is the mid-point of seg JK and the length of seg JL is 3. If K is the midpoint of seg JM, and M is the midpoint of Seg JN, then seg JN =?

Answer

* JK = 3\*2 = 6
* JM = 6\*2 = 12
* JN = 12\*2 = **24**

1. The first three terms of a geometric sequence are k, 6k and 36k. For how many values of k between 1 and 10 inclusive, does the sequence contain only even integers?

Answer

* GP with a=k; r=6
* All terms except 1st are already even
* If 1st term (i.e. k) is even all terms will be even
* K can only take even values {2, 4, 6, 8, 10}
* Solution: **5**

1. If a and b are distinct integers such that ab<1 and b≠0, what is the greatest possible value of a/b?

Answer

* ab<1; so either a or b is negative or a is 0
* if a or b is negative, a/b is also negative
* if a=0, a/b=0
* Solution: **0**

1. A local bank offers its customers two checking account plans:

Plan A: An unlimited number of checks can be written each month for a monthly account maintenance fee of $ 7.50.

Plan B: A monthly account maintenance fee of $2.50 and a transaction fee of $0.50 for each check written during the month.

If Plan A costs a certain customer less than Plan B, what is the least number of checks that this customer writes per month?

Answer

* Calculate breakeven point
* 7.5 = 2.5 + 0.5x
* X = 10 checks
* So for plan a to cost less more than 10 checks must be used
* Solution: **10**

1. For how many ordered pairs of positive integers (a, b) is 5a + 7b < 20?

Answer

* a = 1; b = {1,2}
* a = 2; b = {1}
* Solution: **3**

1. If x and y are positive integers and (x½ y¼)8 = 144, what is the smallest possible value of y –x?

Answer

* x4y2 = 144 = 12\*12 = (2\*2\*2\*2) \* (3\*3)
* x=2; y=3
* Solution: **1**

1. In a certain flower shop, only 3 vases of flowers and 1 wreath can be displayed, in this order, at one time. If there are 10 vases of flowers and 4 wreathes to choose from, how many different arrangements of vases and wreaths are possible?

Answer

* 10C3 3P 4C1 = **2880**

1. It takes 10 people working at the same rate 5 hours to pick 300 apples. How many hours would it take 15 people to pick 900 apples at twice the rate?

Answer

* M1R1T1 = W1
* 10\*R\*5 = 300
* R = 6
* 15\*(2\*6)\*T = 900
* T = **5 hrs.**